IEANTN

Integrated Explicit Analytic Number Theory Network

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October 2026

What analytic number theory is

Using analysis to understand patterns in integers

Where are the primes?

$2 \quad 3 \quad 5 \quad 7 \quad 11 \quad 13 \quad 17 \quad 19 \ldots$

How many primes lie below a given size?

How large can the gaps between them be?

How do integers combine?

$$15 = 3 + 5 + 7$$
$$21 = 3 + 7 + 11$$

Which integers are sums of primes?

How many such representations exist?

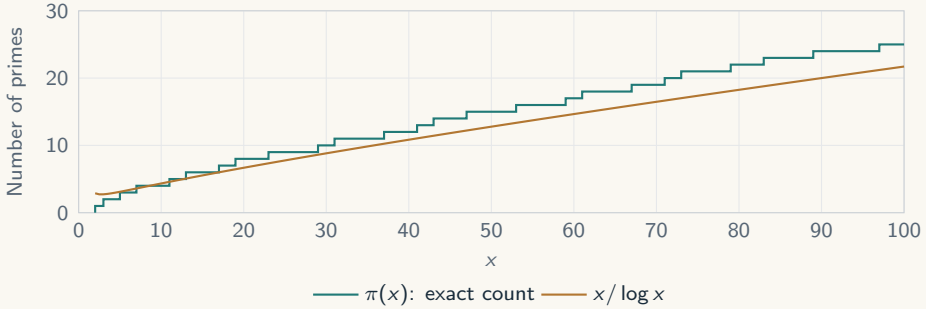
Tools: real and complex analysis, harmonic analysis, probability, ...

The aim is to prove general patterns and estimate how accurate they are.

Usually Big-O is enough

$\pi(x) = \#\{p \leq x : p \text{ prime}\}$

$$\pi(x) \sim \frac{x}{\log x}, \qquad \pi(x) = \frac{x}{\log x} + O\left(\frac{x}{(\log x)^2}\right).$$



Von Koch's equivalence (1901)

Riemann Hypothesis

All nontrivial zeros of $\zeta(s)$ have real part $\frac{1}{2}$.

$$\Updownarrow$$

$$\pi(x) = \mathrm{Li}(x) + \underbrace{O\left(\sqrt{x}\log x\right)}_{\text{error term}} \qquad (x \to \infty)$$

$$\mathrm{Li}(x) := \int_2^x \frac{dt}{\log t}.$$

Sometimes you do need explicit constants

TERNARY GOLDBACH CONJECTURE (now a theorem)

Every odd integer $n > 5$ is a sum of three primes.

Vinogradov's asymptotic theorem

Let $R(n)$ count ordered prime triples $p_1 + p_2 + p_3 = n$. As $n \to \infty$ through odd integers,

$$R(n) = \frac{\mathfrak{S}(n) n^2}{2(\log n)^3} + O\left(\frac{n^2}{(\log n)^4}\right),$$

where

$$\mathfrak{S}(n) = \prod_{p \mid n} \left(1 - \frac{1}{(p-1)^2}\right) \prod_{p \nmid n} \left(1 + \frac{1}{(p-1)^3}\right) \geq 1.$$

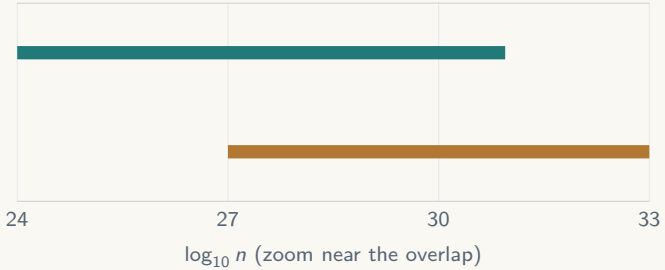
Products over primes. Kumchev–Tolev, Theorem 1, equations (1.3)–(1.4).

Explicit estimates close the gap

Sharp bounds give a threshold we can reach.

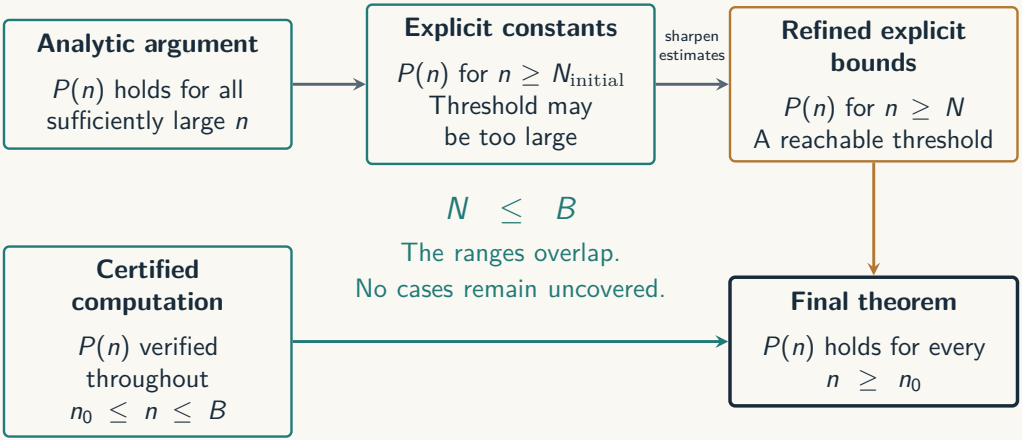
Finite verification
Up to 8.875×10^{30}

Analytic proof
For every $n \geq 10^{27}$



Explicit constants must also be sharp enough. Helfgott (2013/2014) and Helfgott–Platt.

An analytic argument becomes a complete proof

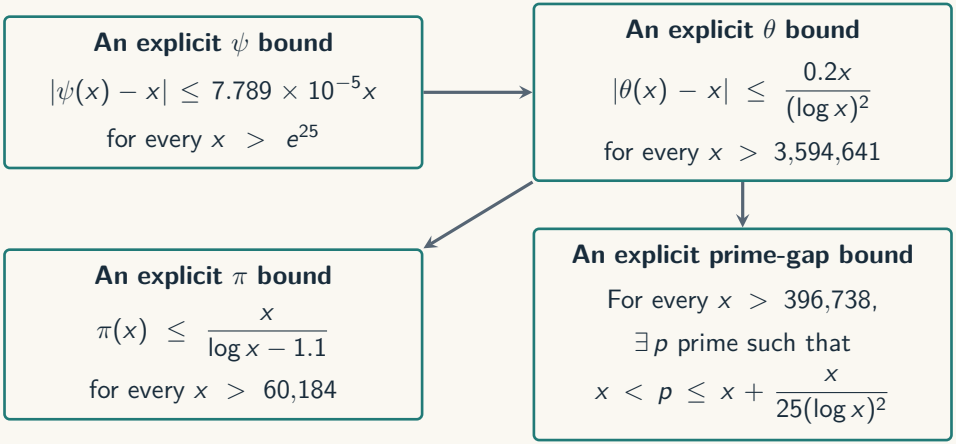


Prime Number Theorem Bookkeeping

Stage	Typical information passed onward
Zeta estimates	Zero-free regions and bounds on zeros
Primary estimates	Error terms for $\psi(x)$
Secondary estimates	Bounds for $\theta(x)$, $\pi(x)$, and prime gaps
Applications	Numerical thresholds in other theorems

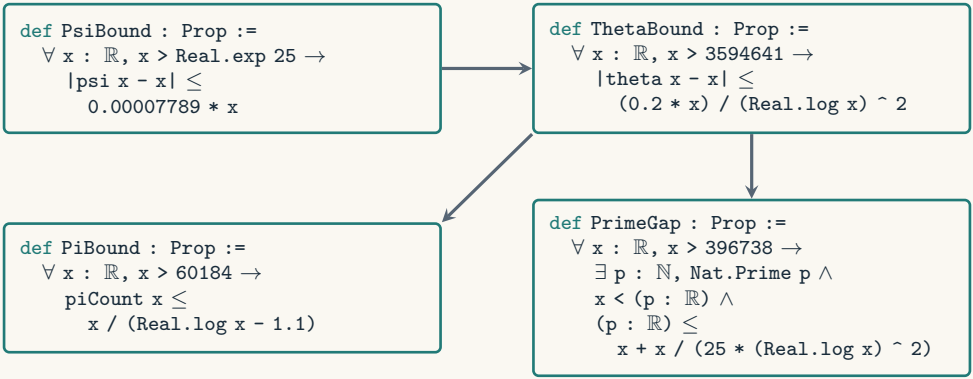
Each new zeta estimate requires revisiting every stage.
Updating the resulting bounds can take years.

Mathematical results form a network



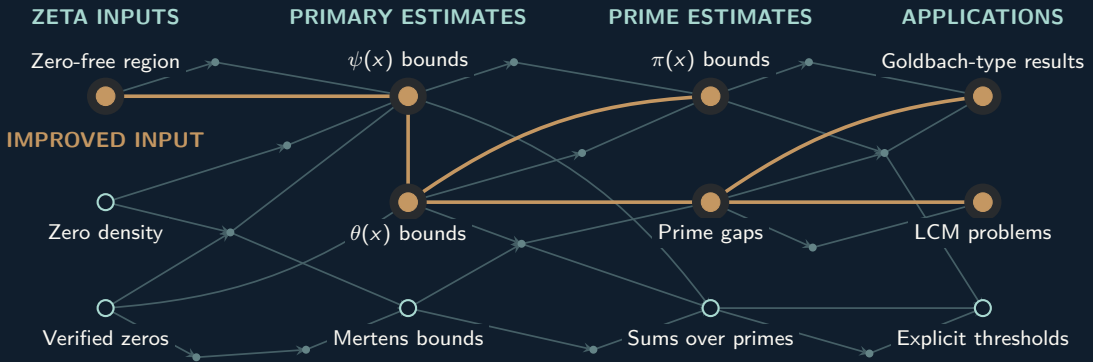
Edges are established conversions (math proofs) in the literature.

The same network, stated in Lean



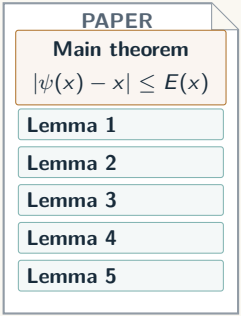
Lean checks proofs. Humans verify that Lean statements agree with the natural language statements.

Goal: a living network



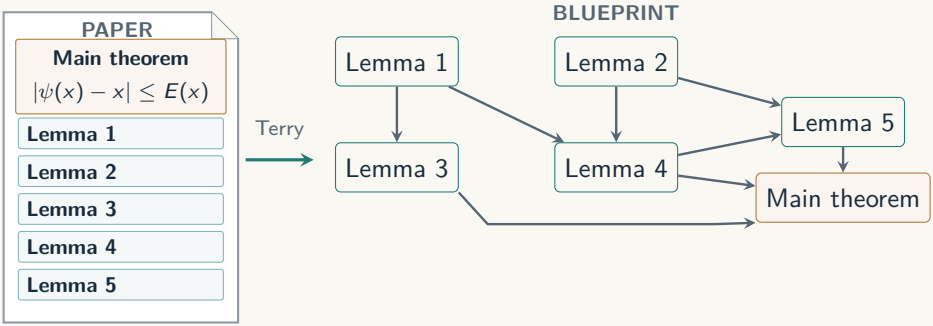
PNT+: papers become a shared blueprint

A math paper on ArXiv



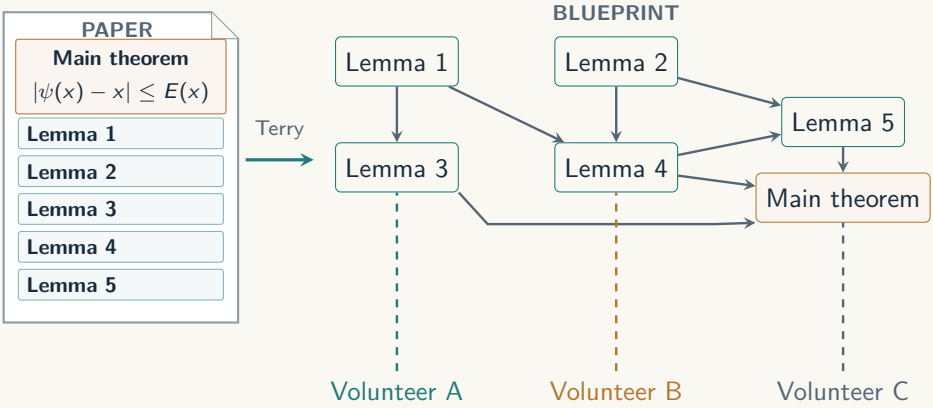
PNT+: papers become a shared blueprint

Terry decomposes the paper into a blueprint



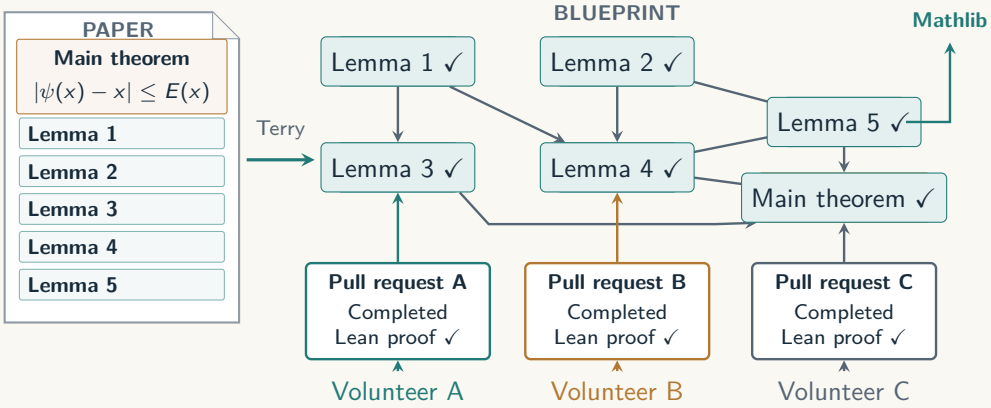
PNT+: papers become a shared blueprint

Volunteers claim different parts of the blueprint.



PNT+: papers become a shared blueprint

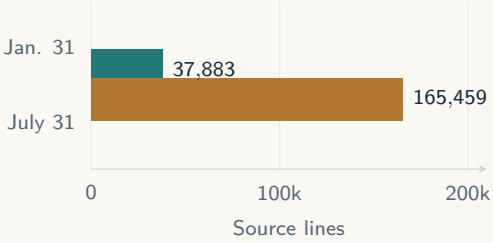
Volunteers submit PRs with the completed LEAN proofs.



The build workload grew with the network

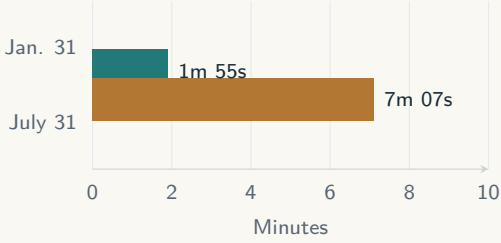
January to July 2026: more source code, more time to build.

Lean source



4.4× as many lines

Clean build time



3.7× as long to build

What changed around June 2026

Generating a proof can be easier than making it reusable and reviewable.

HUMAN-WRITTEN LEAN

proof.lean

```
theorem target (a b c : ℕ) :
  (a + b) + c = a + (b + c) := by
  simpa using Nat.add_assoc a b c
```

Mathlib

Repo lemmas

Find a theorem. Adapt it. Reuse it.

AGENT-GENERATED PR

proof_generated.lean

```
lemma assoc_aux ... := by
  induction c with
  | zero => simp
  | succ c ih =>
    rw [Nat.add_succ, ih]

lemma assoc_variant ... := by
  intro a b c
  induction c with
  | zero => simp
  | succ c ih => ...

theorem target ... := by
  have h1 := assoc_aux ...
  have h2 := assoc_variant ...
```

Reprove. Add a variant. Keep going. . .

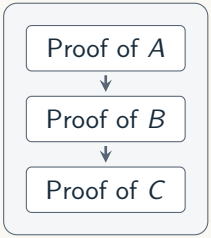
REVIEWER



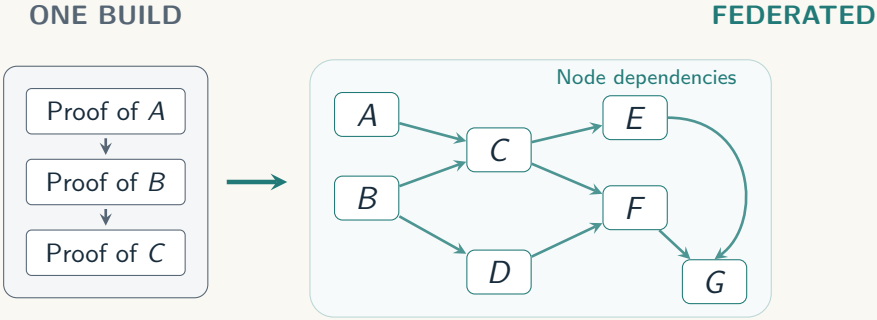
What is new?
What is duplicated?

IEANTN 2.0: a federated network

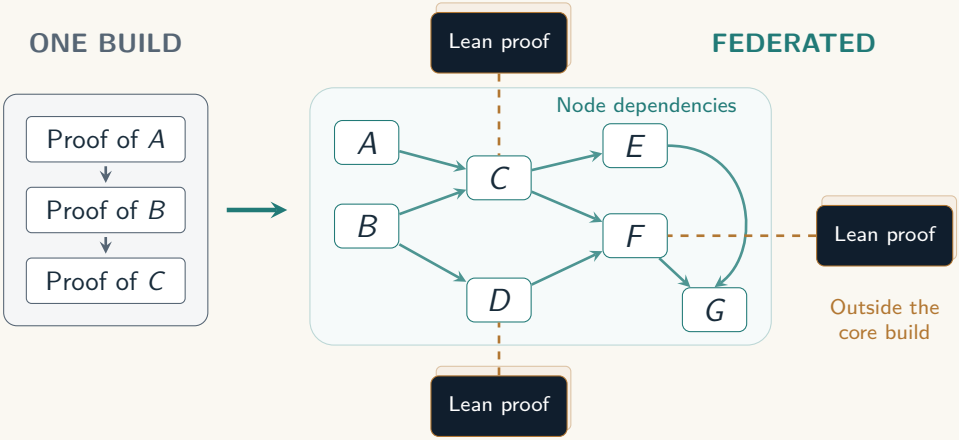
ONE BUILD



IEANTN 2.0: a federated network

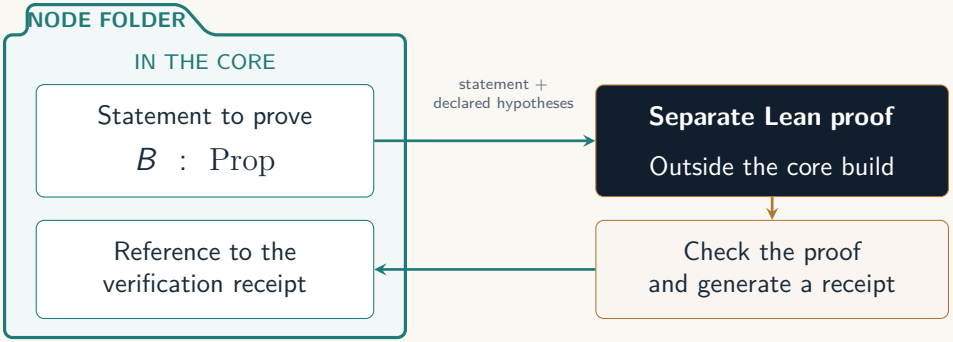


IEANTN 2.0: a federated network



Each theorem lives in a node

A node is a folder containing the statement and its supporting record.



The proof is checked once

Evidence remains visible

`lean-comparator`

A solution has passed the challenge check

`lean-noncompliant`

A Lean development lacks an accepted receipt

`numerical`

External computation supports the claim

`literature`

A cited source provides a proof

`asserted`

A source states the claim without a proof

`bridged`

A checked bridge transports another version's evidence

`none-yet` The claim awaits justification

A concrete dependency: the LCM example

$$L_n = \operatorname{lcm}(1, 2, \ldots, n)$$

"Highly abundant" means $\sigma(N)$ exceeds $\sigma(m)$ for every $m < N$.
Here σ is the sum of positive divisors.

Dusart's prime-gap theorem supplies the construction needed in this proof.

$$n \geq 89{,}693^2 \quad \Longrightarrow \quad L_n \text{ is not highly abundant.}$$

`Lcm.v1`: checked implication. `Dusart2018.v1`: literature input.

A reusable argument exposes its parameter

`Lcm.v1` uses the specific threshold 89,693.

`Lcm.v2` accepts a prime-gap bound starting at X_0.

prime-gap input above X_0

$\Longrightarrow$ L_n is not highly abundant for $n \geq X_0^2$

Subject to $\log X_0 \geq 5$. Both versions can coexist.

The living-network goal

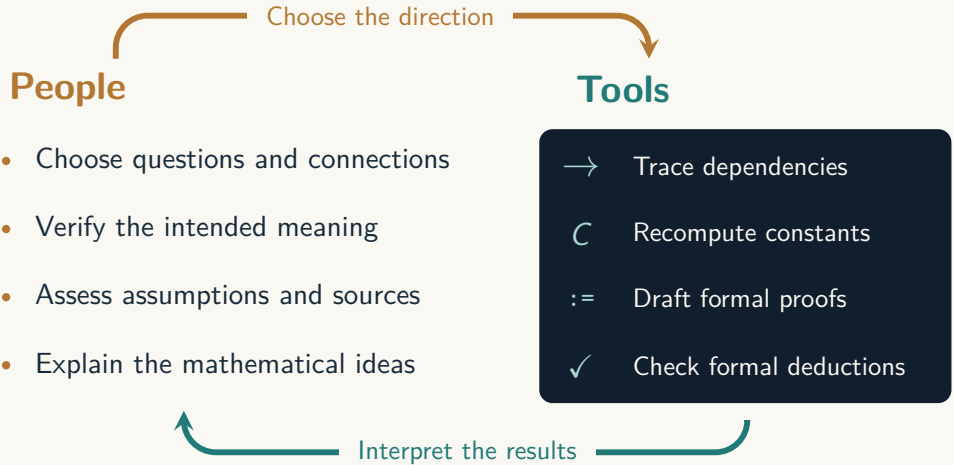
What the architecture already supports

Explicit dependencies, separate LEAN verification, and records of the evidence behind each conclusion

The larger ambition

Update an input and systematically recompute the consequences, with an auditable record

Automation and mathematical judgment



Mathematical understanding remains central

A living network should help us see
how the mathematics fits together.

Machines can carry more of the bookkeeping.

We keep asking what the results mean,
why they matter, and how to explain them.

https://github.com/teorth/IEANTN